Error calculation in tree inspection — You’ve got to be kidding!

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Abstract: Risk assessment of urban trees is becoming more and more sophisticated, not only by using more advanced technical equipment, but also in the application of evaluation methods using measurements to calculate certain properties of trees or assess risk. The use of the slenderness ratio of a tree (tree-height H to breast-height diameter D) is just one example of this trend. However, as soon as something is measured and the results are calculated, error propagation has to be considered. This becomes obvious even in simple applications.

Keywords
› tree measurements
› height to diameter ratio
› error calculation

Introduction
The current trend of using measurable properties of trees for evaluating certain aspects of tree risk seems unavoidable. Contrary to numerical approaches, the ISA Tree Risk Assessment Qualification Program (TRAQ) expresses the probability of failure (of trees or their parts) as well as the likely consequences of failure in one of 4 levels or categories. This approach leaves some flexibility in interpretation.

But, the step before evaluating the probability of failure is often based on measurements and calculations, such as height over diameter or shell-wall-to-radius.

The ratio of tree-height over stem-diameter at breast height seems to be a simple approach. Both values can be measured within seconds and the H/D ratio easily calculated, providing a rating of slenderness as a criteria of stability and safety. But, a closer look into this ‘simple’ task shows that it is actually much more complex than one would expect.

Most of the urban trees we assess for risk are mature. In general, the cross sections of the lower trunk of such trees are elliptical or quite irregular, making measurements difficult and inexact. If you ask 10 different arborists or even experienced risk assessors to measure the diameter of such a large mature tree, you will most likely get 10 different results. In our own trials with experts from all over the world, the average deviation from the mean value was about ±5 to 10%. However, the mean value does not necessarily equal the ‘real’ value because irregularly-shaped cross sections do not have one correct (mean) diameter value.

In addition, although the average
diameter could be determined theoretically, this would not represent the correct value in terms of load-carrying capacity of the cross-section. This is due to the fact that the outer contour (as well as internal wood condition) strongly determines the load that the cross-section can carry. And because this load-carrying capacity of a cross section depends on its diameter to the power of three, even slight changes in diameter and the contour have significant impacts on the result.

Similarly, the height of a tree (H) is often difficult to measure from the ground, even when using modern laser devices. Typical deviations between tree height measurements taken by different practitioners vary by ±5 to 10%, sometimes more.

When risk assessors calculate H/D and draw conclusions about breakage safety from the slenderness ratio (S), they should consider the potential errors. If the mean error of H and D are both ±10%, the mean error of S is ±20%. Errors are additive, depending on how the individual results are combined with each other. This is a consequence of the fact that no one can be sure not having over-estimated one and under-estimated the other value or vice versa.

The same is valid for the shell-wall to radius ratio t/R: error variations of t and R add up as well. Wind-load estimations are more complex. The bending moment at the stem base is proportional to wind speed to the power of two and several other factors. The final error span therefore is a sum of the individual error spans plus two times the error span of wind speed.

Following a publication in the Arboricultural Journal (Mattheck et al. 2002), many experts believe that a slenderness ratio S above 50 is critical in terms of stability and therefore, a justification for removal, even though the stems are intact or lack significant defects. Several forestry experts consider S>70 or even 90 as a critical threshold, depending on species and other factors. Other scientists criticize such thresholds as unverified or based on an incorrect hypothesis (Gruber 2007).

Although controversial, thousands of intact and healthy trees in German cities have been taken down or topped because of a slenderness ratio above 50, and many experts continue to do so. Consequently, this issue needs to be re-evaluated, and threshold policies revised in terms of validity and error spans of assessments.

Regardless of the assumed threshold level (if 50 or 70 or 90), the precision of measurements and the reliability of derived value has to be carefully considered. If, for example, a slenderness value of 55 is calculated, and 50 is seen as the critical threshold, the measurement error described above shows, in all probability, that the real slenderness value of this tree most likely falls between 44 (55-20%) and 66 (55+20%) but may be even lower or higher. Is such a tree safe or not?

The most critical point to understand here is that a single expert measuring one tree one time will never know how the values he obtained differ from the real value. What this means is that one can never be sure if the measured value is much bigger or smaller than the real value of the tree. Consequently, it makes sense to assume the average typical error for a specific assessment, such as ±20% for tree slenderness.

A rational, non-biased expert should therefore recognize that a tree with slenderness S=55 may actually be more dangerous than another tree with S<50, but that the value of 55 is not really significantly above the (controversial) current threshold. Only slenderness values more than the average typical error above the threshold (e.g. 20%) can be considered as significant and reliable enough for drawing conclusions.

Tree assessors should also understand that a span of ±20% does not necessarily mean that the real slenderness value of a particular measured tree falls within this span. An error span of ±20% indicates the average range of deviation, meaning that in individual cases, the error can be bigger or smaller. Therefore, assuming the mean value of repeated measurements is a good approach for the real value and the mean error (=average deviation) is ±20%, some values are outside this range, some inside.

The only way to determine the real value of a natural property as precisely as possible is to have the same property (e.g. tree height) measured by many different experts with different, and pre-calibrated devices. Because this is usually impractical, we have to work with the one measurement by one person, but with the understanding that errors occur unavoidably, in this case with a span of ±20%. However, for other properties of the tree, such as wind load, the typical and unavoidable error span can be significantly higher.

Consequently, evaluations of safety aspects should be carried out and presented carefully and not with digits after the comma that are not proven by accuracy of the measurement. Values should be given with a level of precision proven by the accuracy of the measurement: for example, breast-height-diameter D=50cm (20 inches) instead of D=50.5cm because this level of precision can hardly be achieved at a tree (and irregularly shaped cross sections do not have a certain diameter). In the same way, crown area should not be given in square centimeter (or square inches) but rather in square meters (or square feet) without any digits after the decimal point.

Analyzing the examples outlined above, the mathematical concept of error propagation appears to be much more than just an interesting theory of physicists. Although this issue may not yet be widely recognized, risk assessors should know and understand that, in most cases, there is a certain degree of imprecision involved when they measure even simple properties of trees. And this has to be communicated honestly with colleagues and clients, and it has to be reflected in reports to be considered a true expert.

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Chart 1. (Above) When 10 experts independently measure breast height diameter of the same tree (with an irregularly shaped lower cross-section), this usually leads to 10 different results. This graph shows the ten measurement points and three lines, indicating the mean value as well as the mean ± the average deviation.

Chart 2. (Center) If the span of ± mean percentage deviation is plotted at every measurement point, it becomes obvious that the mean value is not necessarily within this span (purple circles). Using the mean-error span implies that for a significant amount of measurements (=expert assessments) the mean value is not within the span.

Chart 3. (Below) If the span of ± two times the mean percentage deviation is plotted at every measurement point, usually the mean value is within the span. This means, if an expert wants to be safe about having the real value within the given span, he has to take and specify the double mean error span (±).

Literature cited: